New Numerical Methods for Ocean Modeling on Parallel Computers

The Bryan-Cox-Semtner ocean model is a three-dimensional model in Eulerian coordinates (latitude, longitude, and depth). The incompressible Navier-Stokes equations and equations for the transport of temperature and salinity, along with a turbulent eddy viscosity, are solved subject to the hydrostatic and Boussinesq approximations. The model includes a rigid-lid approximation (zero vertical velocity at the ocean surface) to eliminate fast surface waves; the presence of such waves would require use of a very short time step in numerical simulations and hence greatly increase the computational cost. The equations of motion are split into two parts: a set of two-dimensional "barotropic" equations describing the vertically averaged flow, and a set of three-dimensional "baroclinic" equations describing temperature, salinity, and deviation of the horizontal velocity components from the vertically averaged flow. (The vertical velocity component is determined from the constraint of mass conservation.) The barotropic equations contain the fast surface waves and separate them from the rest of the model.

The baroclinic equations are solved explicitly; that is, their solution involves a simple forward time-stepping scheme, which is well suited to parallel computing and presents no difficulty on the Connection Machine. On the other hand, the barotropic equations (two-dimensional sparse-matrix equations linking nearest-neighbor grid points) must be solved implicitly; that is, they must be solved at each time step by iteration. For historical reasons the barotropic equations in the Bryan-Cox-Semtner model are formulated in terms of a stream function. Such a formulation requires solving an additional equation for each island, an equation that links all points around the island. The extra equations create vectorization difficulties when the model is implemented on a Cray and serious communication difficulties when it is implemented on a Connection Machine because a summation around each island is required for every iteration of the implicit solver. Therefore all but the three largest islands had been deleted from the original model, even though eighty islands are resolvable at the horizontal resolution employed (0.5 degrees latitude and longitude). Even so the barotropic part of the code consumes about one-third of the total computing time when the model is executed on a Cray and about two-thirds of the total computing time when the model is executed on a Connection Machine.

The above considerations led us to focus our efforts on speeding up the barotropic part of the code. We developed and implemented two new numerical formulation of the barotropic equations, both of which involve a surface-pressure field rather than a stream function. The surface-pressure formulations have several advantages over the stream-function formulation and are more efficient on both parallel and vector computers.

The first new formulation recasts the barotropic equations in terms of a surface-pressure field but retains the rigid-lid approximation. The surface pressure then represents the pressure that would have to be applied to the surface of the ocean to keep it flat (as if capped by a rigid lid). The barotropic equations must still be solved implicitly, but the boundary conditions are simpler and much easier to implement. In addition, islands then require no additional equations, and therefore any number of islands can be included in the grid at no extra computational cost. Furthermore, and perhaps more important, the surface-pressure, rigid-lid formulation, unlike the stream-function, rigid-lid formulation, exhibits no convergence problems due to steep gradients in the bottom topography. The matrix operator in the surface-pressure formulation is proportional to the depth field H, whereas the matrix operator in the stream-function formulation is proportional to $1/H$. As a result, the latter matrix operator is much more sensitive than the former to rapid variations in the depth of waters over the edges of continental shelves or submerged mountain ranges, where the depth may change from several thousand meters to a few tens of meters within a few grid points. Because such a rapidly varying operator may prevent convergence to a solution, steep gradients were removed from the stream-function formulation by smoothing the depth field. The surface-pressure formulation, on the other hand, converges even in the presence of steep depth gradients. Smoothing of the depth field could significantly affect the accuracy of a numerical simulation of the interaction of a strong current with bottom topography. For example, the detailed course and dynamics of the Antarctic Circumpolar Current (the strongest ocean current in terms of total volume transport) is greatly influenced by its interaction with bottom topography.

As we worked with the surface-pressure, rigid-lid model, we noticed a problem in shallow isolated bays such as the Sea of Japan. In principle, we should have been able to infer the elevation of the ocean surface (relative to the mean elevation) from the predicted surface pressure. We found, however, that the surface heights so inferred were quite different from those expected due to inflow or outflow from the bays. Removing the rigid lid solved that problem, but of course it also brought back the undesirable and unneeded surface waves. We were able to overcome that new difficulty by treating the terms responsible for the surface waves implicitly, which artificially slows down the waves, whereas the rigid-lid approximation artificially speeds up the waves to infinite velocity. (Either departure from reality is acceptable: Climate modeling does not require an accurate representation of the waves because they have little effect on the ocean circulation.)

Those considerations led us next to abandon the rigid-lid approximation in favor of a free-surface formulation. The surface pressure is then proportional to the mass of water above a reference level near the surface. The benefits of the surface-pressure, free-surface model are greater physical realism and faster convergence of the barotropic solver. In particular, the revised barotropic part of the code, including eighty islands, is many times faster than the original, including only three islands (when both are implemented on the 0.5-degree grid). In addition, the surface pressure is now a prognostic variable that may be compared to global satellite observations of surface elevation to validate the model, and satellite data may now be assimilated into the model to improve short-term prediction of near-surface ocean conditions.

None of our revisions, of course, changed the fact that the large matrix equation in the barotropic solver must be solved implicitly. We chose to use conjugate-gradient methods for that purpose because they are both effective and easily adapted to parallel computing. Conjugate-gradient methods are most effective when the matrix is symmetric. Unfortunately, the presence of Coriolis terms (terms associated with the rotation of the earth) in the barotropic equations makes the matrix nonsymmetric. By using an approximate factorization method to split off the Coriolis terms, we retained the accuracy of the time-discretization of the Coriolis terms and produced a symmetric matrix to which a standard conjugate-gradient method may be applied. We also developed a new preconditioning method for use on massively parallel computers that is very effective at accelerating the convergence of the conjugate-gradient solution. The method exploits the idea of a local approximate inverse to find a symmetric preconditioning matrix. Calculating the preconditioner is relatively expensive but need be done only once for a given computational grid.

Further Reading

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